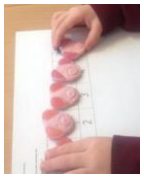
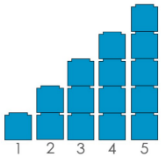
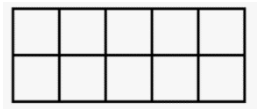
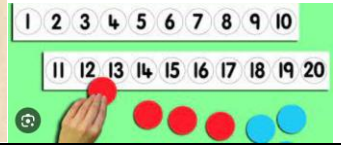
Year N – Multiplication			
There are no requirements for children to multiply or divide during nursery due to their developmental stage and cognitive ability. However, children learn the importance of key concepts such as “groups, lots, none” and “what two looks like” which are the foundations for them to be able to multiply and divide in the future.	Concrete **Building tables** Children explore lots of. “stuff” in every context imaginable! Adults and children count in units of one initially to three and then five. They do this in songs and rhymes and with a huge range of different objects.   **Grouping** Children compare groups of one and ‘not one’; lots. They count and later subitise groups of two moveable things. **Arrays** Children explore arrays in practical scenarios such as cookery, mud kitchen and malleable play.  **Doubles** Children begin to compare groups, knowing when two groups are the same.	Pictorial **Building tables** Children place single objects into models depicting bigger units such as numicon, five frames and Hungarian dice models.  Children are shown numbers as they say the number names. Children are shown numbers regularly which relate to an amount of objects.  Children are shown moveable pictures.  **Grouping** Stories are used and children look at unmoveable images of quantities.	Abstract **Building tables/ Grouping/ Arrays/ Doubles** Adults use everyday, simplistic language relating to key concepts. Adults pose questions and interact in ways which evoke children to use this language in relation to objects and pictures they can see.
Year N - Division			

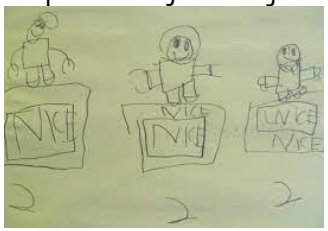
<table>
<tr>
<td>Children learn key concepts such as grouping, comparing, same, fair and sharing as pre-requisites to conceptually understanding division.</td>
<td><u>Grouping/ Sharing</u>
Children explore sharing a group of objects between two and more people.

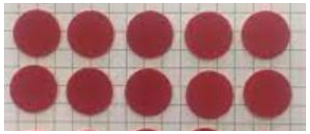
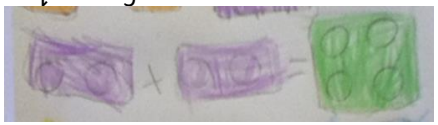
This is a daily occurrence during play with toys and objects. Children also share during snack time, for example sharing fruit, allowing three breadsticks each etc. They are taught systematic ways of doing this as part of nursery routines.
  </td>
<td><u>Grouping/ Sharing</u>
Children move images to share such as cards during a board game.

Some children explore graphics – showing how a set has been shared.
  </td>
<td><u>Grouping/ Sharing</u>
Adults model using everyday language of sharing groups. Children begin to use this language.</td>
</tr>
<tr>
<td colspan="4">Year R - Multiplication</td>
</tr>
<tr>
<td>Early Learning Goals:
• Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.
• Have a deep understanding of number to 10, including the</td>
<td>Concrete
<u>Building tables</u>
 
Children will first count in units of one. This is practised on a daily basis to ensure children can consistently do it forwards and backwards to 10 and later 20. Staircase arrangements are used to support this.
 
Towards the end of the year children will begin to explore the concept of unitising through skip counting in 2s initially up to 6. Use practical equipment to support their</td>
<td>Pictoral
<u>Building tables</u>

Children will be shown how the objects relate to numerals. For example, there are two socks and two more socks relating to the numerals 2 and 4.
They will look at number tracks to support them in this area. They will use ten frames which are set out in columns of two.
 
 
  </td>
<td>Abstract
<u>Building tables</u>

Adults model talking about models and pictorial representations to see patterns and make links. Adults model stem sentences.
 
There are _____ grapes in each bunch.
There are _____ bunches
The are ____ grapes altogether.</td>
</tr>
</table>

composition of each number; this could be that a number is made up of groups of two etc. • Subitise quantities; This could be seeing groups of numbers within numbers.	understanding so that they can begin to count forwards and back in 2s. Models that come in pairs such as shoes and gloves or pairs of children. Counting sticks and moveable tracks in which carpeted number can be moved and objects placed on top. Songs and rhymes are good ways to reinforce counting in 2s.   **Grouping** Children start by using a variety of objects to help their understanding of equal groupings. Children explore the concepts of same/ different in relation to objects and the groups of objects. They understand how to recognise whether they are equal. They describe how many are altogether by counting all of the objects. Children explore already created arrays. The re able to place moveable objects into model arrays.	Children begin to record their ideas of quantities units through graphics. This can be pictures of the objects representing units.  They will write numbers to represent number tracks with units of 1 and may begin to record units of 2. **Grouping** Children compare images of objects. They recognise which are the same and which are different.  They draw and recognise equal groups. They describe how they know when a group is equal using the language of amounts. 	**Grouping** Adults model and then children describe equal groups use number words and words such as, “same, different, more, less, fewer.” They create stem sentences to describe their groups. There are _____ altogether. There are _____ groups in total There are _____ in each group

	  **Arrays** As above then children progress to copying/ imitating the arrays they have encountered. Create arrays through use of counters/ concrete equipment. Children use practical resources.   **Doubles** Children use concrete objects to show doubles – double 3 = 6. Use objects in pairs to demonstrate doubles. Songs and rhymes such as, "Double, double dumpling" support understanding.	**Arrays** Children begin to use graphics to show arrays. They draw pictorially onto images of numicon/ arrays. They notice groups within a quantity.  **Doubles** Use pictorial representations to show doubles. Activities such as playing board games using dice support this. 	**Arrays** Describe arrays through seeing patterns. "I can see two, two and two." **Doubles** Recalling knowledge of doubles.

			
Year Reception - Division	Concrete	Pictorial	Abstract
Early Learning Goals • Have a deep understanding of number to 10, including the composition of each number; this could be that a number is made up of groups of two etc. • Subitise quantities; This could be seeing groups of numbers within numbers.	**<u>Grouping</u>** Children explore creating two **equal** groups from a whole. They practically sort a whole set of objects into two equal groups. Children could sort counters into sharing hoops or share objects between two puppets. They understand that equal means each group has the same amount. They are shown how to do this systematically.  **<u>Sharing</u>** Children explore sharing a set of objects into equal parts as a model of division. They work out, using practical equipment how many are in each part. Children need to be	**<u>Grouping/ Sharing</u>** Children use graphics/ pictorial representation to explore sharing a set into two equal groups.	**<u>Grouping/ Sharing</u>** Adults model and children use everyday language to describe how they have shared a set into two equal groups.

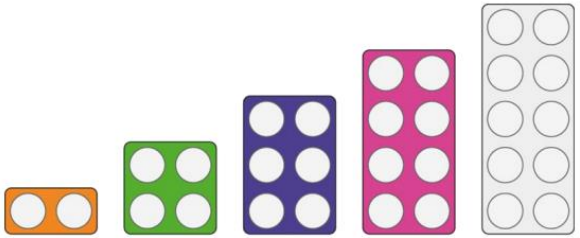
	given the opportunity to see when a number of objects cannot be shared equally.		
Year 1 – Multiplication			
	Concrete	Pictorial	Abstract

National Curriculum objectives

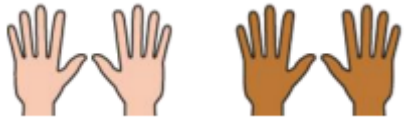
- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations, and arrays with the support of the teacher.
- Non-statutory - Through grouping and sharing small quantities, pupils begin to understand multiplication and division; doubling numbers and quantities; and finding simple fractions of

Building tables

Children will begin to explore the concept of unitising through skip counting in 2s up to 20, moving up to 50 once they are confident. Use practical equipment to support their understanding, ensure that they count forwards and back in 2s. The counting stick, songs and rhymes are good ways to reinforce counting in 2s.



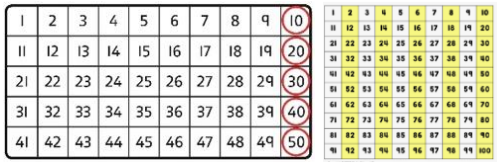
Move into counting in 10s, forwards and backwards so children become more fluent. Children could use hands to represent naturally counting in10.



Building tables

Use of counting stick to write steps of 2s,5s and 10s.

Children will find the total of equal groups by counting in 2s, 5s and 10s
100 squares, number lines and the Gattegno chart may be useful to support counting in 2s, 5s and 10s. Children will establish patterns.

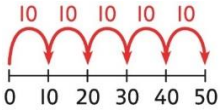


Children will create links with 2s and even numbers. Children to count forwards and backwards.

When counting in 2s children will develop the understanding of how this may be used in real life contexts, such as counting in pairs. Helping them see when it is more efficient than counting on ones. How many wheels are there? Count in groups of two.

Building tables

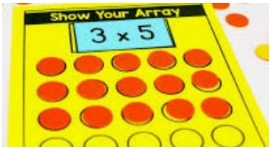
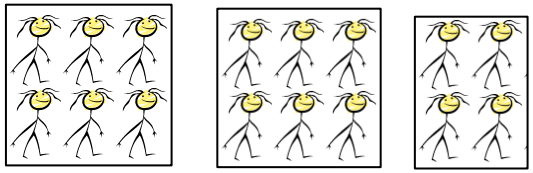
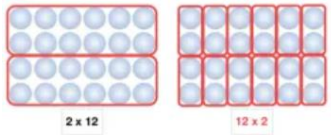
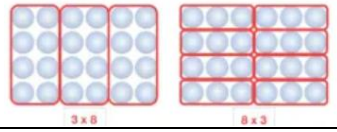
Finding the total of equal groups by counting in 2s, 5s and 10s
Use a number line to support repeated addition through counting in 2s, 5s and 10s.

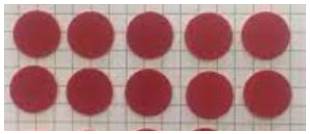
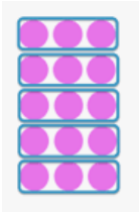
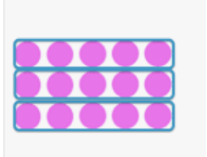
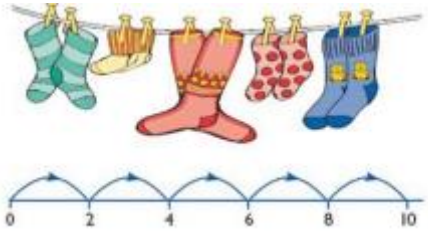


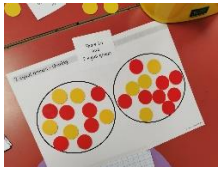
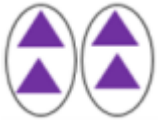
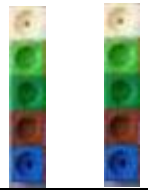
Use stem sentences alongside pictorial representations to see patterns and make links –

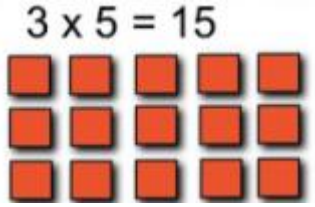
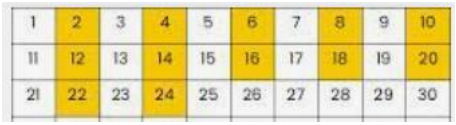
There are _____ grapes in each bunch.
There are _____ bunches
The are ____ grapes altogether.

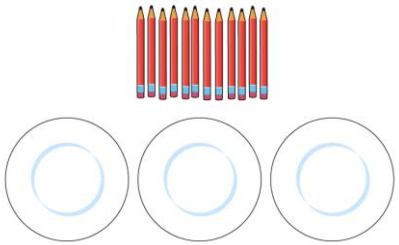
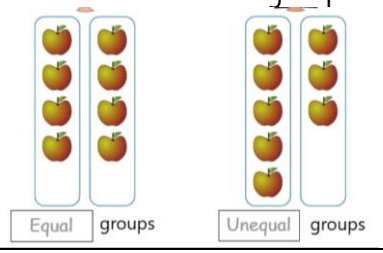
objects, numbers and quantities. • They make connections between arrays, number patterns, and counting in twos, fives and tens.	Finally, move into counting in 5s, children could use fingers to represent natural way to count in 5s.  Use of counting stick to help with counting in twos (to 24), fives (to 50) and tens (to 100). Children will find the total of equal groups by counting in 2s, 5s and 10s using concrete apparatus.  There are 5 pens in each pack … 5…10…15…20…25…		

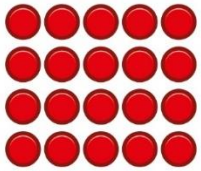
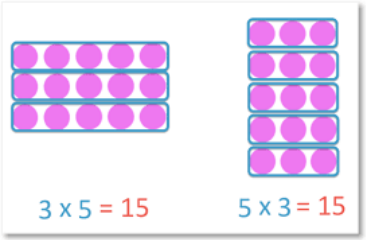
	Grouping Children start by using stories to help their understanding of equal groupings. They understand how to recognise whether they are equal. They describe how many are altogether, how many groups in total and how many in each group. Children progress into creating arrays with concrete objects to show they are a rectangular arrangement to show the equal groups. There are 3 groups of 5, there are 15 groups in total.  	**Grouping** Children draw and recognise equal groups. They describe how they know when a group is equal.   Progress into drawing arrays to represent equal groups   	**Grouping** Children describe equal groups in words: Two equal groups of four Four equal groups of two They create stem sentences to describe their groups. There are _____ altogether There are _____ groups in total There are _____ in each group

	Arrays Create arrays through use of counters/ concrete equipment. Children use practical resources (peg boards and counters are useful) to make equal groups in columns or rows.  	**Arrays** Draw arrays and show pictorial representations to support their understanding. They develop links between arrays and equal groups. There are 3 counters in each row. There are 5 rows. 3 + 3 + 3 + 3 + 3 = 15 	**Arrays** Describe arrays through seeing patterns. They can write stem sentences linked to columns or rows. There are 3 rows of 5 counters 5 + 5 + 5 = 15 in total. There are 5 columns of 3 counters 3 + 3 + 3 = 15 in total I 
	Doubles Use concrete objects to show doubles – double 3 = 6. Use objects in pairs to demonstrate doubles. 	**Doubles** Use pictorial representations to show doubles.  	**Doubles** Solve problems involving doubles. You have five pairs of socks, how many socks do you have in total?

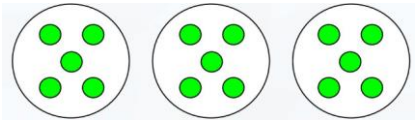
Year 1 – Division			
National Curriculum objectives	Concrete	Pictorial	Abstract
Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	**Grouping** Children explore creating **equal** groups from a whole. They practically sort a whole set of objects into equal groups. Children could sort counters into sharing hoops or use multilink to create arrays. They understand that equal means each group has the same amount and begin to see that sometimes a whole cannot be shared amongst equal groups.  	**Grouping** Children use pictorial representation to explore creating equal groups. There are 4 altogether. There are 2 equal groups of 2. 	**Grouping** Children write sentences and solve problems using words. I have 20 cubes. How many equal groups of 5 can I make? How many equal groups of 2 an I make?
	Sharing Children explore sharing a set of objects (e.g. multilink) into equal parts as a model of division. They work out, using practical equipment how many are in each part. Children need to be given the opportunity to see when a number of objects cannot be shared equally. 	**Sharing** Use pictorial representations to represent sharing into equal parts. I have 12 sweets to share with 3 people. If I share equally, each person will get 4 sweets. 	**Sharing** Children will problem solve with sharing using stem sentences to describe sharing. There are ____ altogether. They are shared equally between ____ groups. There are ____ in each group. They will also explain how the know when a situation occurs where they can't share equally.

Year 2 – Multiplication			
National Curriculum objectives	Concrete	Pictorial	Abstract
• recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers • calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs • show that multiplication of two numbers can be done in any	**Build tables**  Develop an understanding of how to unitise groups of 2, 5 and 10 and learn corresponding times-table facts. Use arrays to develop understanding with concrete objects.  5 x 3 = 15 3 x 5 = 15 *3 groups of 5 … 5, 10, 15* *3 × 5 = 15*	**Build tables**  Understand how to relate counting in unitised groups and repeated addition with knowing key times-table facts. Create pictorial representations of multiplication through arrays.  Use 100 squares to look for patterns within the times tables. 	**Build tables**  See patterns in 2, 5 and 10 times tables. Understand how times tables increase and contain patterns. Create fact families from times tables 5 x 2 = 10 2 x 5 = 10
	Grouping Children use concrete equipment, such as counters, multilink or objects and then show them being grouped. They make links to this as repeated addition. Use	**Grouping** Move into pictorial representations. Draw rings around the resulting groups to clearly demarcate them. Ensure that whilst exploring these early stages of multiplication, a circle or ring is used	**Grouping** Describe groups through creating equations. Make patterns between repeated addition and multiplication.

order (commutative) and division of one number by another cannot • solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.	drawn circles or paper plates to physically move the objects into the groups.  Before grouping – there are some **pencils** There are twelve pencils in total.  After grouping – The pencils have been grouped. There are three groups of 4. 4 + 4 + 4 = 12 Introduce children to the x symbol and help them to create links between repeated addition and multiplication.	to enclose each group, referring to the previous separate objects now as one 'thing/group'.  Before grouping – there are some footballs. There are 6 footballs in total.  The footballs have been grouped. There are three groups of 2. 2 + 2 + 2 = 6 in total. Ensure the children are exposed to pictorial representations with unequal amounts. The children should recognise that one group has a different amount. Help children to use visual representations to compare equal and unequal groups.  	*5 + 5 + 5 = 15* *3 x 5 = 15* *I know that 5 + 5 + 5 = 15* *So I know that 3 groups of 5 or 3 x 5 = 15*

	Arrays and commutativity Use arrays to help support the understanding of multiplication, creating links between repeated addition, arrays and multiplication.  I have 4 groups of 6 6 + 6 + 6 + 6 = 24 4 x 6 = 24 Use arrays with concrete materials to visualise commutativity linked to relevant times tables. I can see 6 groups of 4. I can see 4 groups of 6. 4 x 6 = 24 6 x 4 = 24 6 + 6 + 6 + 6 = 24 4 + 4 + 4 + 4 + 4 + 4 = 24	**Arrays and commutativity** Children draw arrays to help support the understanding of multiplication. They link it to their learning on grouping.  *4 groups of 5 … 5 groups of 5* Use arrays to visualise commutativity. Rotate the array to show that orientation does not change the multiplication.  *This is 3 groups of 5 and also 5 groups of 3.*	**Arrays and commutativity** Children understand the relationship between arrays, multiplication and repeated addition. Create equations to describe arrays 3 x 5 = 15 3 + 3 + 3 + 3 + 3 = 15 3 groups of 5 = 15 altogether They describe commutativity through words, showing understanding of links with multiplication and repeated addition. I know that 3 x 5 = 15 So, I know that 5 x 3 = 15. So, I know 5 + 5 + 5 = 15 I also know 3 + 3 + 3 + 3 + 3= 15

<table>
<tr><td colspan="4">Year 2 – Division</td></tr>
<tr><td rowspan="2">National Curriculum objectives</td><td><u>Sharing using groups</u>
Use counters and concrete materials to develop understanding of sharing for numbers less than 20. Children physically share the total, for example between paper plates or with sharing circles.

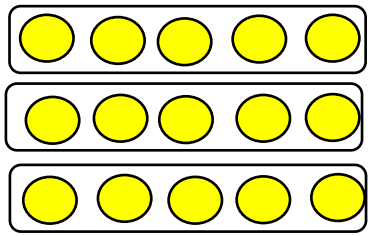
 
15 counters shared between 3. They get 5 each. Look at when groups are shared equally and when there are leftovers.</td><td><u>Sharing using groups</u>
Use pictorial representations to show sharing in groups for numbers less than 20.
 
10 muffins shared between 5 people – how many do they each get?

Use representations such as bar models to help show the total shared into equal parts.

<table><tr><td colspan="5">10</td></tr><tr><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td></tr></table></td><td><u>Sharing using groups</u>
Look at contextual problems involving sharing.

There are 7 cakes and 2 children. How many cakes will they each get? ‘Leftovers’ may be introduced.

There are 15 sweets in a bag. How many children can have 5 each?</td></tr>
<tr><td><u>Use known facts</u>
 
Build understanding of times table knowledge and using these known facts to develop a relationship between multiplication facts and division. Create links through counting stick activity.

Create groups/ arrays with concrete materials. 15 counters shared into 3 groups. 15 divided by 3 is 5.
  </td><td><u>Use known facts</u>
 
Build understanding of times tables knowledge to link equal grouping with repeated subtraction and known times-table facts to support division.

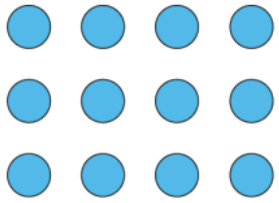
Use a bar model to support understanding of the link between times-table knowledge and division.

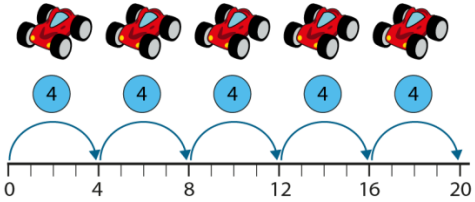
<table><tr><td colspan="5">15</td></tr><tr><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td></tr></table>
<table><tr><td colspan="3">15</td></tr><tr><td>5</td><td>5</td><td>5</td></tr></table></td><td><u>Use known facts</u>
 
Relate times-table knowledge directly to division. Create fact families for multiplication and division. Use knowledge of their times table to create facts.

I know that 3 groups of 5 = 15
So, I know that 15 divided by 5 = 3
I also know that 5 groups of 3 = 15
So, I know that 15 divided by 3 = 5</td></tr>
</table>

<table>
<tr><td colspan="4">Year 3 – Multiplication</td></tr>
<tr>
<td rowspan="2">National Curriculum objectives
• recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
• write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
• solve problems, including missing number</td>
<td><u>Building tables</u>
Recall 10,5, 2 and learn 4-, 3- and 8-times table through use of counting stick and practical activities. They build on their knowledge of groups to learn and embed their times tables.

Links are created to the 6-, 8- and 12-times table through repeated doubling.

Children will create arrays using counters and other concrete apparatus. Exploring links between doubles and times tables and looking for patterns. Ensuring that children understand that multiplication is commutative.
 
4 x 3 = 12 or 3 x 4 = 12</td>
<td><u>Building tables</u>
Recall through pictorial representations times tables.
 

How many wheels are there altogether?
 

Children count in groups of 4.
Use the mathematical language of product and factors when describing the equation: -
What is the product of 4 and 5
4 is a factor
5 is a factor
20 is the product of four and five.</td>
<td><u>Building tables</u>
Recall through mental maths. Create fact families and establish links.

If I know 6 x 5 = 30
I know 5 x 6 = 30

Look for strategies and describe patterns in times tables.

Reinforce the mathematical vocabulary of product and factor.

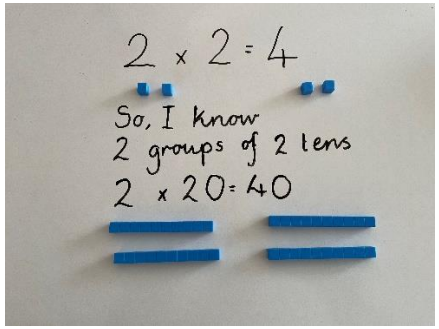
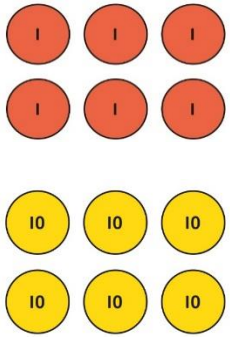
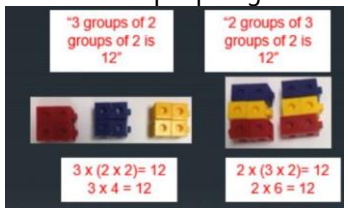
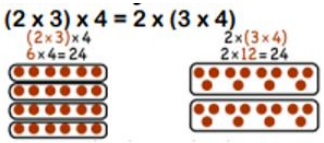
Introduce missing number questions.

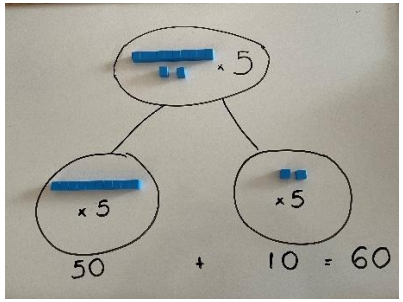
If I know that the product of 3 x (a) is 12. How can I use this to find what a is worth?</td>
</tr>
<tr>
<td><u>Known related facts</u>

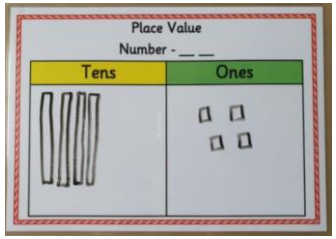
Using place value equipment explore the relationship between known times-tables and multiples of 10. Ensure that children understand how unitising can support us.</td>
<td><u>Known related facts</u>

Use pictorial representations to continue to understand how unitising 10s supports multiplying by multiples of 10.</td>
<td><u>Known related facts</u>

Understand how to use known times-tables to multiply multiples of 10. Understand commutative law and</td>
</tr>
</table>

problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects		 *3 groups of 2 ones is 6 ones.* *3 groups of 2 tens is 6 tens.* *6 tens is ten times bigger than 6 ones.*	create fact families from these known facts. 3 x 2 = 6 3 x 20 = 60 6 = 3 x 2 60 = 3 x 20 Include missing number calculation __ x 3 = 60 If I know that 3 x 2 is 6 – how can this help me with to find the missing number? What other known facts can I find? What is the same? What is different?
	Associative property Use counters/ materials to explore associative property 	**Associative property** Use pictorial representations to explore associative property. 	**Associative property** Create equations to show the relationship pattern (2x3) x 4 = 2 x (3x4) Find missing numbers to balance equations. (2x3) x ? = 2 x (? x 4)
	Partitioning Partition counters/ objects to show informal recording of partitioned numbers to multiply with friendlier numbers. e.g. 12 x 5 10 x 5 = 50 2 x 5 = 10 50 + 10 = 60	**Partitioning** Partition through pictorial methods such as the use of part whole model to show informal recording of partitioned numbers Use place value charts and pictorial representations to show how to link	**Partitioning** Write equations with addition to solve calculations of multiplying 1 digit by 2 digit. 4 x 24 = 4 x 20 = 80 4 x 4 = 16 80 + 16 = 96

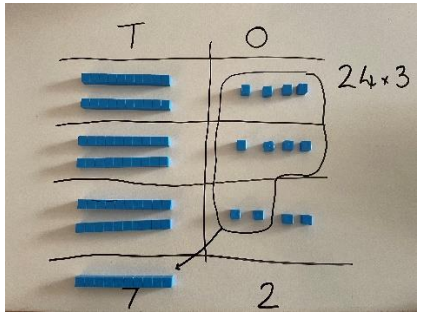
<table>
<tr>
<td></td>
<td> 
</td>
<td>multiplying a two digit by a one-digit number – e.g. 2 x 12.

 
</td>
<td>3 x 24 = 72

Include missing number calculation and show equations in various forms.

___ x 5 = 60 ____ = 10 + ___

10 x 5 = 50 ___ x 5 = 10
50 + 10 + 60</td>
</tr>
<tr>
<td></td>
<td><u>Multiplication of 2-digit number by a 1-digit number (expanded method)</u>
Use place value equipment, showing when appropriate how 10 ones are exchanged for one ten

 
</td>
<td><u>Multiplication of 2-digit number by a 1-digit number (expanded method)</u>
Use pictorial representations with place value charts progressing into the grid method when ready.

3 x 24

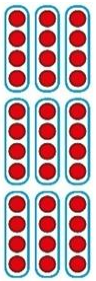
<table><tr><td>X</td><td>3</td></tr><tr><td>20</td><td>60</td></tr><tr><td>4</td><td>12</td></tr></table></td>
<td><u>Multiplication of 2-digit number by a 1-digit number (expanded method)</u>
Expanded version
24
x 3
12 (3 x 4)
60 (3 x 20)
72

Leading to compact method when ready, ensuring children are confident with how to exchange when needed.

24
x 3
72
1</td>
</tr>
</table>

<table>
<tr><td colspan="4">Year 3 – Division</td></tr>
<tr><td rowspan="2">National Curriculum objectives
recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
solve problems, including missing number problems, involving multiplication</td><td><u>Build tables</u>
 
Use concrete resources to divide into equal groups. Link this to knowledge of known times-tables to calculate divisions. Use language linked to division with counting stick.

18 counters divided into groups of 6 – 3 groups of 6 counters. 18 ÷ 3 = 6.</td><td><u>Build tables</u>
 
Use pictorial resources to help create links with knowledge of known times-tables.

 
36 ÷ 4 = 9
There are 9 groups of 4

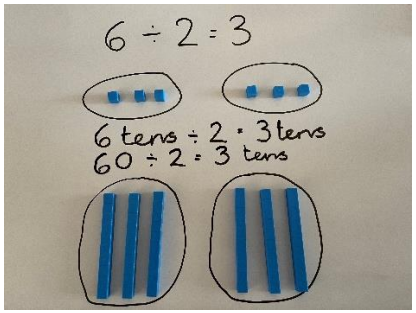
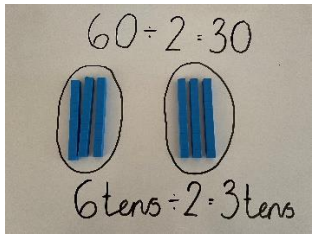
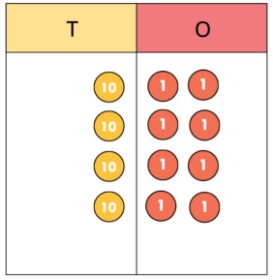
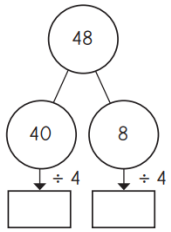
Bar model representations to show dividing into equal groups. Reinforce that multiplication is commutative.

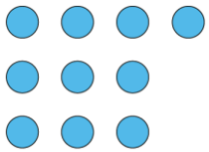
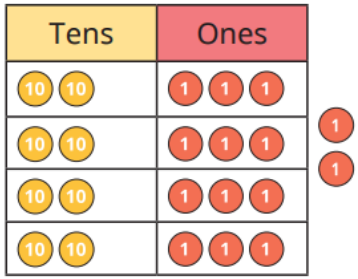
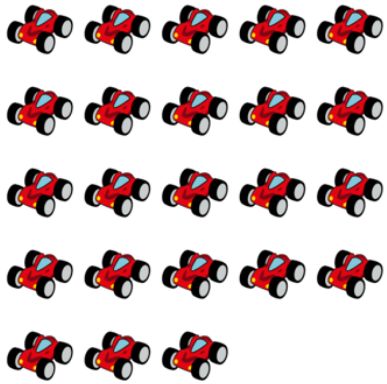
<table><tr><td colspan="9">36</td></tr><tr><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td></tr></table>
<table><tr><td colspan="4">36</td></tr><tr><td>9</td><td>9</td><td>9</td><td>9</td></tr></table></td><td><u>Build tables</u>
 
Children write STEM sentences to show their understanding of how their times-tables knowledge helps them to calculate divisions.

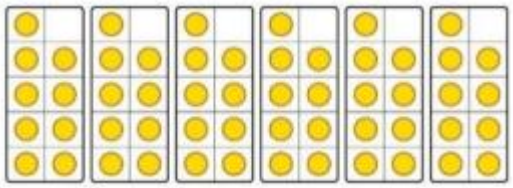
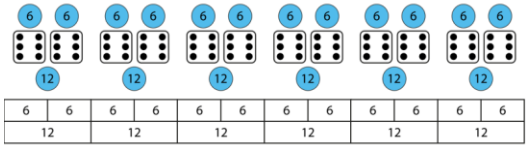
I need to work out 36 cakes shared between 4.

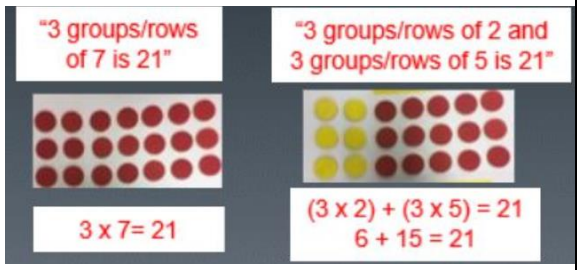
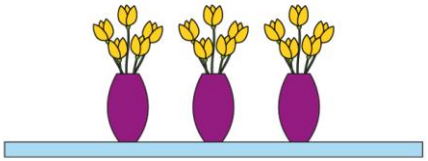
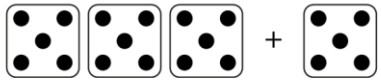
I know that 4 x 9 = 36
so I know that 36 ÷ 4 = 9.

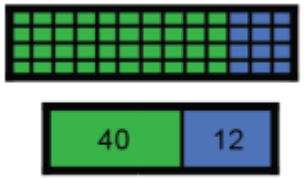
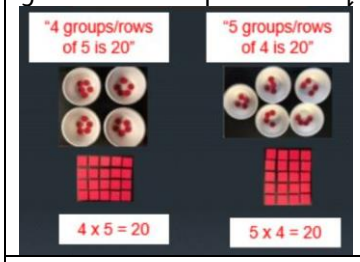
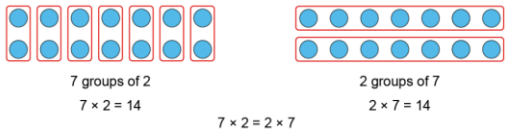
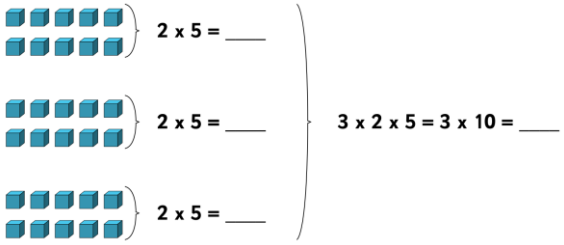
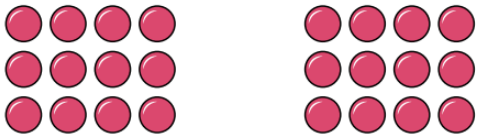
Children understand how division is related to both repeated subtraction and repeated addition.</td></tr>
<tr><td><u>Known facts</u>
Children use concrete resources, for example place value equipment to explore how to divide by unitising.</td><td><u>Known facts</u>
Children use pictorial resources to help understand dividing by unitising. They divide multiples of 10 by unitising.</td><td><u>Known facts</u>
Creating links for division through use of times tables, dividing multiples of 10 by a single digit. They can articulate their understanding</td></tr>
</table>

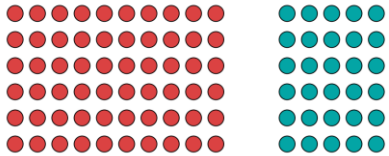
and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects	If I know that 6 divided by 2 = 3 I know that 6 tens divided by 2 = 3 tens 60 ÷ 2 = 30 	 *If I have 6 tens shared into 2 equal groups.* 3 tens in each group. 3 tens = 30	through the use of STEM sentences, in particular that the answer is 10 times smaller. 150 ÷ 3 = ? 150 is 15 tens. 15 divided by 3 is 5. 15 tens divided by 3 is 5 tens. 15 ÷ 3 = 5 150 ÷ 3 = 50 What is the same? What is different?
	Division without remainders Children use concrete apparatus, such as place value charts and counters or dienes to explore dividing 2-digit numbers by a one-digit number.  48 ÷ 4 = 12	**Division without remainders** Children use pictorial representations to explore dividing without remainders. They may use a part-whole model and partition into friendly numbers to complete the division. 40 ÷ 4 = 10 8 ÷ 4 = 2 10 + 2 = 12 48 ÷ 4 = 	**Division without remainders** Children solve problems involving division and can articulate their answer through STEM sentences. Sarah thinks that 55 sweets can be shared equally between 5 people. Is she correct? Convince me. I have partitioned ____ into ____ tens and _____ ones.

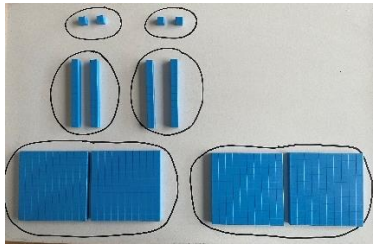
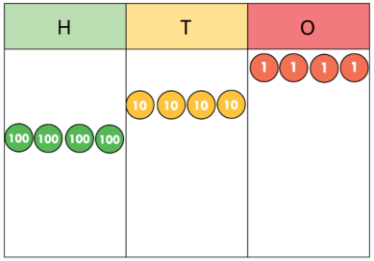
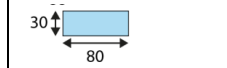
	Dividing with remainders. Children use concrete equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further and there are objects left over.  *There are 10 counters in total.* *There are 3 groups of 3, with 1 remainder.* *Start with smaller numbers to understand the concept and then begin to explore larger numbers.* 94 ÷ 4 =  	**Dividing with remainders** Use pictorial representations to explain remainders.  There are 23 cars to share between 5 children. *23 ÷ 5 = 4 remainder 3*	**Dividing with remainders** Understand that the remainder is what cannot be shared equally from a set. Children use STEM sentences to show their deeper knowledge with the use of times table knowledge to find how many groups the amount can be shared into. 23 ÷ 5 = ? 3 × 5 = 15 4 × 5 = 20 5 × 5 = 25 … this is larger than 22 So, 22 ÷ 5 = 4 remainder 3

Year 4 – Multiplication & Division			
Multiplication			
National curriculum	**Concrete**	**Pictorial**	**Abstract**
• recall multiplication and division facts for multiplication tables up to 12 × 12 • use place value, known and derived facts to multiply and divide mentally, including multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers • recognise and use factor pairs and commutativity in mental calculations • multiply two-digit and	**Building tables**  Building tables, for example, build tables using counting stick, forwards and backwards and with missing jumps as you progress. To be able to recall all times tables learnt so far and learn the 6, 7-, 9-, 11- and 12-times tables. Explore links between times tables, use arrays with counters to help children establish links. Give children time to explore patterns with concrete resources. Understand the special cases of multiplying by 1 and 0.  0 + 0 + 0 3 × 0	**Building tables** Represent the relationship between different tables through pictorial representations e.g. see the relationship between 9 and 10 times table.  Or the relationship between 10-, 11- and 12-times tables Create links between other times tables using pictorial representations. 	**Building tables** Understand patterns between times tables. Be able to articulate the links between the ×3 table, ×6 table and ×9 table Use techniques to help learn patterns with more difficult tables. ×5 table and ×7 table 4 × 7 = 4 × 5 + 4 × 2 ×9 table and ×10 table 7 x 10 = 70 7 x 9 = 70 – 7 = 63 X 12 table 4 x 12 = 4 x 10 + 4 x 2 Use these links to help solve problems.
	Using known facts Use place value counters to show known facts. Unitise with counters. If 3 x 5 = 15 then 3 x 500 = 1,500 and 1,500 ÷ 3 = 500.	**Using known facts** Use pictorial representations and unitising to show known facts. If 3 x 5 = 15 then 3 x 500 = 1500 and 1,500 ÷ 3 = 500	**Using known facts** Using known facts to simplify some multiplications. Children look for friendlier numbers to work with when multiplying.

three-digit numbers by a one-digit number using formal written layout • solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects			$24 \times 5 = 12 \times 2 \times 5$ $12 \times 2 \times 5 =$ $12 \times 10 = 120$ So, $24 \times 5 = 120$
	Distributive property of multiplication Ensure children explore the distributive law. Use counters and concrete apparatus to create equations to show distributive law.  Multilink could also be used to help children understand. Distributivity 3 x (2 +4) = 3 x 2 + 3 x 4 So the '3' can be 'distributed' across the '2 + 4' into 3 times 2 and 3 times  4 leading to 13 x 4 = 10 x 4 + 3 x 4 = 52	**Distributive property of multiplication** Use pictorial representations to explore understanding of distributive law.  3 × 5 = 15 5 × 3 = 15 Write two multiplication equations to show the diagram. 3 x 5 = 15 and 5 x 3 = 15. What is the same and what is different? Both equations have factors of 3 and 5. Both equations have product of 15. As they become more secure in understanding show them how it can be distributed across and still has the same answer.  3 × 5 + 1 × 5 = 4 × 5	**Distributive property of multiplication** Pupils should be able to represent such relationships using mixed operation equations, for example: $5 \times 6 = 4 \times 6 + 6$ or $5 \times 6 = 4 \times 6 + 1 \times 6$ $4 \times 6 = 5 \times 6 - 6$ or $4 \times 6 = 5 \times 6 - 1 \times 6$ Use an array to write multiplication sentences and reinforce repeated addition.  2 + 2 + 2 + 2 = 8 4 + 4 = 8 2 x 4 = 8 4 x 2 = 8 Children balance equations to find missing equations 4 x 5 = 2 x 5 + __ x 5

			
	Commutative law Reinforce children's understanding of commutive law. Use counters to apply the commutative property of multiplication. Create arrays using counters/ cubes to generate multiplication equations. 	**Commutative law** Draw arrays in different rotations to find commutative multiplication sentences and to show those connections. For example, that 7 groups of 2 and 2 groups of 7 both correspond to the same total quantity 	**Commutative law** Create fact families (understanding that multiplication is commutative but not division) Create relationships. 8 x 4 = 32 4 x 8 = 32
	Multiplying 3 numbers Multiply 3 numbers using the associative law. Create arrays using counters to show it does not matter which order you multiply in. Ensure that they make links with commutativity and look at grouping numbers more efficiently. 	**Multiplying 3 numbers** Use pictorial representations to multiply 3 numbers.  4 x 3 x 2 Explore making groups more efficient.	**Multiplying 3 numbers** Use times table knowledge to multiply 3 numbers. Look for known facts that can help with multiplying 3 numbers. 6 x 5 x 2 = 6 x 10 = 60

	5 x 3 x 2 = (5 x 3) x 2 = 15 x 2 = 30 **or** **5 x 3 x 2 = 5 x (3 x 2) = 5 x 6 = 30**		
	Partitioning Make multiplications by partitioning into friendly numbers, use concrete apparatus to support. **15 x 6** **10 x 6** **5 x 6** 	**Partitioning** Use pictorial representations to understand how multiplication can be completed through partitioning into numbers that are easier to work with.	**Partitioning** Partition numbers to multiply with friendly numbers. 14 x 6 = 10 x 6 + 4 x 6 = 60 + 24 = 84 14 x 6 = 84 Solve missing number equations.
	Short multiplication Use place value equipment to make multiplications. Make 6 x 212 using equipment  There are 6 x 2 ones = 12 ones There are 6 x 6 tens = 36 tens There are 6 x 2 hundreds = 12 hundreds 1,200 + 360 + 12 = 1,572	**Short multiplication - Grid method** (If needed for conceptual understanding) | | 300 | 50 | 2 | |---|---|---|---| | 5 | | | | Use grid method or place value counters alongside a column method. Start with two digits x 1 digit and progress up to 3 digits x 1. 352 x 5 = Short multiplication	**Short multiplication Expanded** (Begin with 2-digit number by a one-digit number and progress into 3 digits x 1 digit) Understand how the expanded column method is related to the formal column method and understand how any exchanges are related to place value at each stage of the calculation. 36 x 44 36 x 4 24 (4 x 6) + 120 (4 x 30) 144

			<u>Into formal method –</u> once children are secure with exchanges and how they are related to place value at each stage of the calculation. 36 X 4 144
	<u>Multiplying by 10 and 100 and multiples of 10 and 100</u> Children use unitising with concrete apparatus to create links between multiplying by 10 and 100. I can see 2 groups of 2 ones = 4 I can see 2 groups of 2 tens = 40 I can see 2 groups of 2 hundreds = 400 	**<u>Multiplying by 10 and 100 and multiples of 10 and 100</u>** Children use pictorial representations with unitising to create links by multiplying by 10 and 100. Children understand that when using a place value chart to multiply multiples of 10 and 100 that the amount becomes 10 times or 100 bigger.  4 x 10 = 40 40 x 10 = 400 4 x 100 = 400 Once confident children can look at multiplying look at real life problems that	**<u>Multiplying by 10 and 100 and multiples of 10 and 100</u>** Children find missing numbers and create links. ____ x 10 = 500 5 x ____ = 5,000 Children solve worded problems. Eggs come in boxes of thirty. A supermarket orders eighty boxes of eggs in one week. How many eggs does the farmer need to supply?  80 boxes of 30 eggs: $30 \times 80 = 3 \times 8 \times 10 \times 10$ $= 3 \times 8 \times 100$ $= 2.400$

will help them multiply more than one number.

'There are ten sweets in one bag. If four children have three bags of sweets each, how many sweets do they have altogether?'

1,000s	100s	10s	1s	
			4	× 10
		4	0	
			3	× 10
		3	0	
		1	2	× 10
	1	2	0	× 10
1	2	0	0	

ten times the size | ten times the size | ten times the size

$6 \times 50 = \square$ $\qquad$ $70 \times 80 = \square$

$\square = 4 \times 50$ $\qquad$ $80 \times 50 = \square$

$\square = 30 \times 60$ $\qquad$ $90 \times \square = 10{,}800$

$70 \times \square = 3{,}500$ $\qquad$ $\square 0 \times \square 0 = 3{,}600$

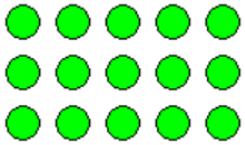
Year 4 – Division

- Recall multiplication and division facts for multiplication tables up to 12 x 12
- Use place value, known and derived facts to divide mentally
- Divide two-digit and three-digit numbers by a one-digit number using formal written layout (not explicitly stated in the programme of study but implied in the non-statutory guidance)

Concrete

Link to tables

For example, use language of division linked to tables using counting stick.



Use objects to explore fact families.
5 x 3 = 15
15 ÷ 5 = 3

Use known facts

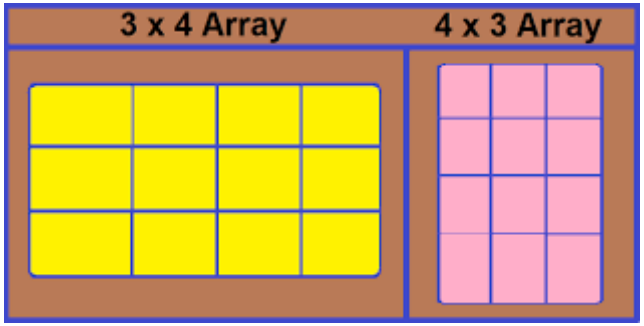
Create using concrete apparatus to understand how unitising and known facts can help with division.

If 6 ÷ 3 = 2, then 60 ÷ 3 = 20 and 600 ÷ 3 = 200

Pictorial

Link to tables

Represent division using arrays and through pictorial representations.



Children will be familiar with 3 x 4 or 4 x 3 = 12. Looking at the array differently reveals the inverse, that is 12÷3=4 or 12 put into 3 rows makes 4 columns – or 4 in each row.

Use known facts

Create pictorial representations for known facts. Use unitising to support working with bigger numbers.

Abstract

Link to tables

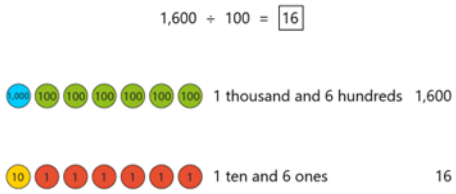
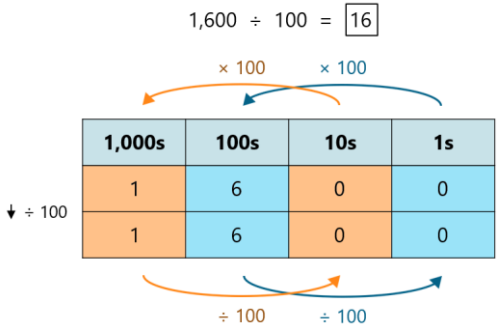
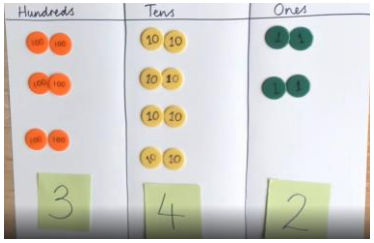
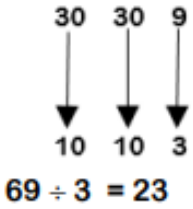
Explore patterns through understand families of related multiplication and division facts. I know that 5 × 4 = 20, so I know all these facts:

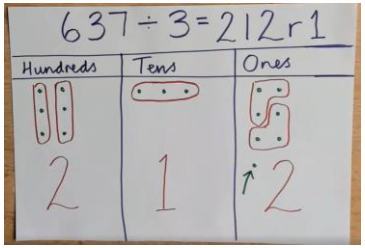
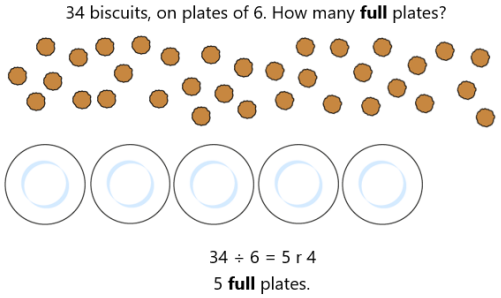
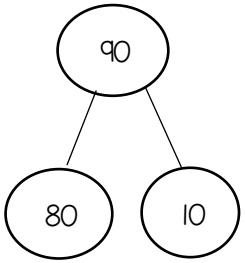
5 x 4 = 20
4 x 5 = 20
20 = 4 x 5
20 = 5 x 4
20 ÷ 5 = 4
20 ÷ 4 = 5
5 = 20 ÷ 4
4 = 20 ÷ 5

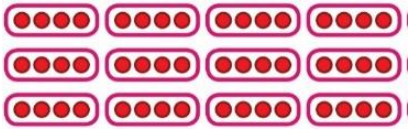
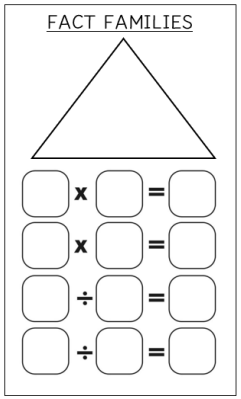
Find missing number facts using what I know.

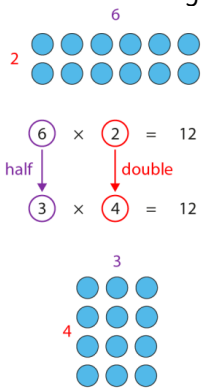
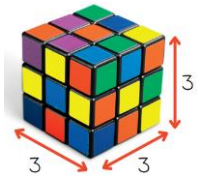
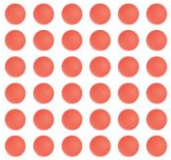
Use known facts

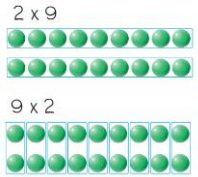
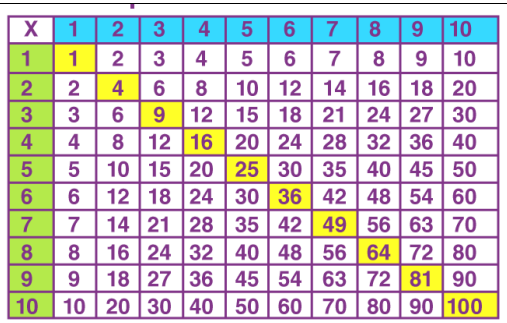
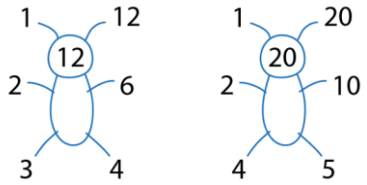
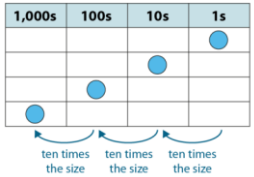
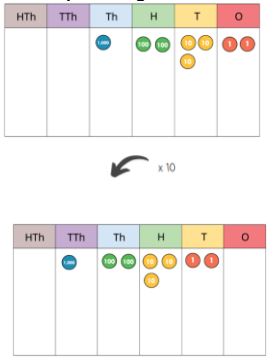
Pupils must also be able to apply their automatic recall of multiplication table facts to solve division problems, for example, solving 28 ÷ 7 = 4 , by recalling that 4 x 7 = 28 . Understanding that 280 ÷ 7 = 40. They use this to help with division of 10s and 100s by a single digit.

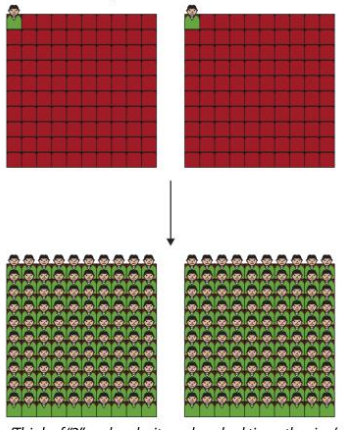
	Dividing by 10 and 100 and seeing relationships by dividing by multiples of 10 and 100 Children use place value counters to help them create relationships between dividing between 10 and 100. Children understand how their knowledge of unitising can help them. 	**Dividing by 10 and 100 and seeing relationships by dividing by multiples of 10 and 100** Children use pictorial resources to help understanding of dividing by 10 and 100. Ensure children understand that the answer is 10 or 100 times smaller.  1,000s 100s 10s 1s 1 6 0 0 1 6 0 0	**Dividing by 10 and 100 and seeing relationships by dividing by multiples of 10 and 100** Children using their knowledge to help them with known facts. If I know 12 ÷ 3 = 4 I know 120 ÷ 3 = 40 I also know that 1,200 ÷ 3 = 400. I can also used known facts to help me find missing numbers. _____ ÷ 500 = 50 _____ ÷ 1,200 = 120
	Dividing without remainders Children use materials and place value chart to divide 2 and 3-digit numbers by 1 digit number. They partition into hundreds, tens and ones and show how to divide dividends into equal groups. 684 ÷ 2 	**Dividing without remainders** Rearrange the dividend to find multiples of the divisor. 69 ÷ 3 (what do I know about the 3 times table? Partition my 60 into 30 + 30 -  69 ÷ 3 = 23 or Partition into tens and ones 60 ÷ 3 = 20 9 ÷ 3 = 6	**Dividing without remainders** *Use known facts to help with dividing* **Short division** 372 ÷ 6 = 6 ⟌ 3 7 12 (6 2) 372 divided by 6. 3 hundreds cannot be shared equally between 6, so exchange the hundred for 30 tens. I know have 37 tens which shared equally by 6 is 6 with a remainder of 1 ten. Exchange the ten for 10 ones. I now have 12 ones which shared equally between 6 is 2. The answer is 62.

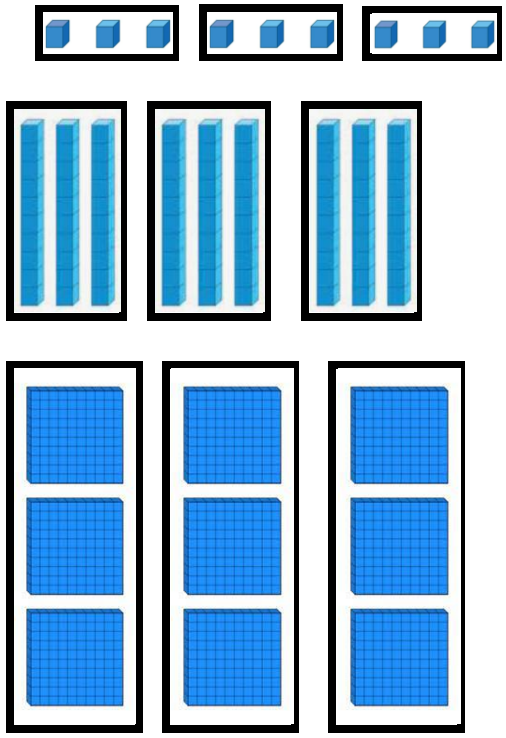
		Represent how to partition flexibly where needed to create more effective calculations.	
	Dividing with remainders Use concrete materials and place value equipment to explore remainders. Show remainder as how many are left that cannot be shared by the divisor. 47 ÷ 4 =  	**Dividing with remainders** Represent through arrays, showing the remainder as the part that cannot be shared equally or through pictorial representation on a place value chart.  Children use pictorial representations to help dividing with remainders.  	**Dividing with remainders** Understand how partitioning can help with remainders. Children use their times table knowledge to partition into friendly  90 ÷4 80 ÷ 4 = 20 10 ÷ 4 = 2 90 ÷ 4 = 22 r 2

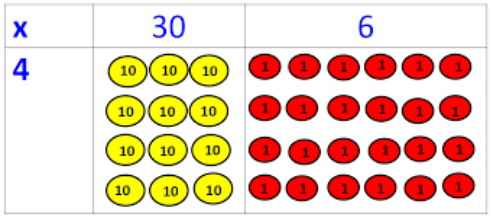
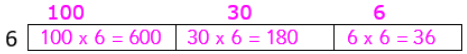
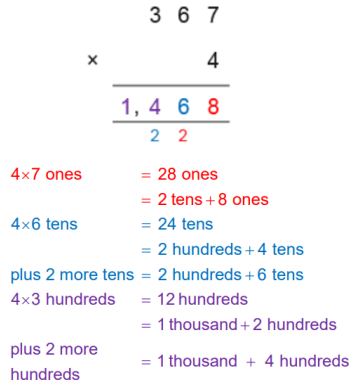
Year 5 – Multiplication			
National curriculum	**Concrete**	**Pictorial**	**Abstract**
• identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers • know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers • establish whether a number up to 100 is prime and recall prime numbers up to 19 • multiply numbers up to 4 digits by	**Building tables and create links to inverse**  Continue to ensure children are exposed to times tables for multiplication facts for multiplication tables up to 12 x 12. Use times table knowledge to apply to multiples of 10, 100 and 1,000 using counting stick- forwards and backwards and with missing jumps. Use concrete equipment where needed to group and share and to explore the calculations that are present. Understand unitising to understand multiplying in multiples of 10, 100 and 1,000. I have 21 counters. I made 7 groups of 3. There are 21 in total. I have 21 in total. I shared them equally into 7 groups. There are 3 in each group. I have 21 in total. I made groups of 3. There are 7 equal groups. 3 x 7 = 21. If I know 3 x 7 = 21 I know that 30 x 70 = 2,100	**Building tables and create links to inverse**  Use pictorial representation to embed multiplicative relationships and explore fact families to understand division facts.  4 x 12 = 48 12 x 4 = 48 48 ÷ 4 = 12 48 ÷ 12 = 4 Use pictorial resources to help explore these fact families. 	**Building tables and create links to inverse** Understand missing number problems for multiplication calculations and know how to solve them using inverse operations. 2 x ? = 22 ? x 2 = 22 Children use their knowledge of links between times tables to create relationships between if they double one number then they must half the other to obtain the same product. Alternatively, if they multiply one factor then they must divide the other factor by the same for the product to stay the same. Create STEM sentences to show their knowledge. I know that if I multiply one factor by 2 then I must divide the other factor by 2 for the product to stay the same.

a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers • multiply and divide numbers mentally, drawing upon known facts • divide numbers up to 4 digits by one-digit number using the formal written method of short division and interpret remainders appropriately for the context • multiply and divide whole numbers and		Explore links between times tables using pictorial representation, for example the effect of halving one factor and doubling the other. If children half one factor and double another, the product will stay the same. 	
	<u>Understanding factors,</u> Children use concrete resources, such as counters, multilink and cubes to understand the meaning of square and cube numbers. Children should understand that a square number is the result of a number multiplied by itself. For example, 4 rows of 4 = 16, 16 is a square number. They explore cube numbers using cubes and multilink. 27 is a cube number 3 x 3 x 3 	**<u>Understanding factors</u>** Use images to explore examples and non-examples of square numbers. 6 x 6 = 36 6 ² = 36  Children look for patterns when finding square numbers.	**<u>Understanding factors</u>** Use multiplication tables to look for patterns in finding common multiples, common factors, square, prime numbers, and cubed numbers. Can children spot a pattern? Children can identify which are prime and composite numbers and explain how they know. Children complete STEM sentences to show their knowledge. I know that the cube of a number is the result of multiplying the number by ___ and then ____ again.

those involving decimals by 10, 100 and 1,000 • recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³) • solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares, and cubes • solve problems involving addition, subtraction, multiplication and division	Use counters to create arrays to explore factors of a number e.g., factors of 18. Use the counters to find all the factors.  Explore using concrete apparatus the rules with factors for numbers (they always have pairs except for when they are a prime number). Ensure children become confident in finding all factor pairs, understanding how to work in a systematic way.	 They understand that if a number cannot multiply a whole number by itself to create it then it is not a square number. Use pictorial representations to find factor pairs. 	I know that ____ is a multiple of _____ so it is a factor of _____.
	Multiplying by multiples of 10, 100 and 1,000 Use place value equipment to explore multiplying by unitising, allowing children to move from additive to multiplicative thinking. Use place value charts and counters to show how when you multiply by 10 it becomes ten times the size. 	**Multiplying by multiples of 10, 100 and 1,000** Use pictorial resources such as place value charts to explore how to multiply by multiples of 10, 100 and 1,000. 	**Multiplying by multiples of 10, 100 and 1,000** Explore efficient methods and see patterns with using known facts and unitising to multiply. Explore how can find out what you don't know using these known facts. Solve problems using these methods. If I know 3 × 6 = 30 Then I know 3 × 60 = 300 I also know 3 x 600 = 4,000

and a combination of these, including understanding the meaning of the equals sign • solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates	2 x 1 = 2 ones = 2 2 x 10 = 2 tens = 20 2 x 100 = 2 hundreds 200 2 x 1,000 = 2 thousands = 2,000	Including understanding the effect of repeated multiplication by 10. Children see the relationship between multiplying by 100 and multiplying a number by 10 and then 10 again. Use pictorial representations to help develop understanding of conceptual scenarios. There were 2 people in the cinema this morning. This afternoon there are 100 more people, how many people are there this afternoon?  2 x 100 = 200 Two hundred is one hundred times as many as two people. There are two hundred people in the cinema this evening.	Children write STEM sentences to show their understanding. To multiply by 100, I can multiply by 10 and 10 again. To multiply by 1,000 I can multiply by 10 x 10 x 10. Children use their understanding of multiplying by 10,100 and 1,000 and using the inverse to help them find missing numbers. ____ x 1,000 = 15,000
	<u>Known facts</u> Use concrete apparatus and unitising to use known facts to find other facts. I know that 3 x 3 = 9	**<u>Known facts</u>** Use place value equipment and knowledge of unitising to represent what I know to find what I don't know.	**<u>Known facts</u>** Make links using known facts and unitising to multiply. 5 × 5 = 25 5 × 50 = 250

	30 x 3 = 90 300 x 3 = 900 	2 x 4 = 8 2 x 40 = 80 2 x 400 = 800	5 × 500 = 2,500 5 × 5,000 = 25,000 Understand all the relative facts through working systematically. 5,000 × 5 = 25,000 Find missing numbers using known facts and the inverse. 5 x ____ = 2,500
	Short multiplication - multiplying up to 4-digit numbers by a single digit Use concrete resources to explore how to multiply efficiently exploring partitioning to work with friendlier numbers.	**Short multiplication - multiplying up to 4-digit numbers by a single digit** Use pictorial representations to show partitioning numbers to multiply efficiently. Partition into relevant place value amounts. Use an area model or place value chart and then and then add the parts.	**Short multiplication - multiplying up to 4-digit numbers by a single digit** If needed use expanded column multiplication initially. Once secure move into compact column multiplication, ensuring to include any required exchanges.

	Place value counters with an area model chart or place value chart would be a good way to explore with concrete apparatus. 36 x 4 = 144 30 x 4 = 120 4 x 6 = 24 		
	<u>Long multiplication - multiplying up to 4 digits by 2 numbers</u> Partition one number into place value amounts thinking about working with friendlier numbers e.g., 10s and 1s and then add the parts. Use the grid method to break it down. Start with 2 digits by 2 digits and work way up to 4 digits by 2 digits. Use place value charts to support. 14 x 13 = 182 10 x 10 = 100 4 x 10 = 40 3 x 10 = 30 3 x 4 = 12	**<u>Long multiplication - multiplying up to 4 digits by 2 numbers</u>** Use grid method to break down when introducing multiplying 2 digits by 2 digits. Then add the parts together. *125 × 12 = 1,500*	**<u>Long multiplication - Multiplying up to 4 digits by 2 numbers</u>** Use column multiplication, children will need to be secure in their understanding of 0 as a place holder. Begin with expanded form ensuring understanding of place value at each stage.

	100	*20*	*5*
10	*1,000*	*200*	*50*
2	*200*	*40*	*10*

		2	1	9	0
x				6	9
	1	**9**	**7**	**1**	**0**
1	**3**	**1**	**4**	**0**	**0**
1	**5**	**1**	**1**	**1**	**0**

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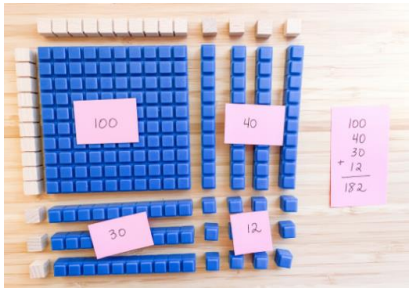
<td>Use an area model with concrete apparatus to explore this method.

 
</td>
<td>Show children expanded column method, starting with 2 digits by 2 digit and working way up to 4 digits by 2 digits.</td>
<td>Progress to include examples that require multiple exchanges as understanding, confidence, and fluency build.</td>
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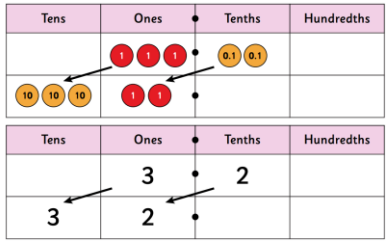
<td><u>Multiplying decimals by 10,100 and 1,000</u>
Use place value charts and counters for children to move counters along the number of places needed to the left. Use counters to model the exchange of 10 tenths, 10 hundredths or 10 thousandths. Ensure misconceptions are addressed with adding zero and children understand how much each one is increasing by e.g. x 10 – it is ten times bigger.

3.2 x 10 = 32
 
</td>
<td><u>Multiplying decimals by 10,100 and 1,000</u>
Use pictorial representation on a place value chart to help represent multiplication by 10,100 or 1000.
 

0·37 x 10 = 3.7</td>
<td><u>Multiplying decimals by 10,100 and 1,000</u>
Use STEM sentences to articulate understanding, see patterns between the multiplication and understand that to multiply by 100 you could multiply by 10 and then 10 again.

0.37 x 10 = 3.7
0.37 x 100 = 37
0.37 x 1,000 = 370

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<tr><td></td><td></td><td></td><td>0</td><td>.</td><td>3</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>3</td><td>.</td><td>7</td><td></td></tr>
<tr><td></td><td></td><td>3</td><td>7</td><td>.</td><td>0</td><td></td></tr>
<tr><td></td><td>3</td><td>7</td><td>0</td><td>.</td><td>0</td><td></td></tr>
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Once secure solve problems involving these.</td>
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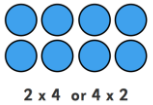
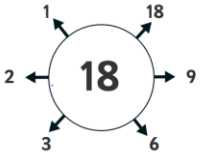
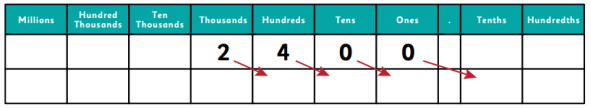
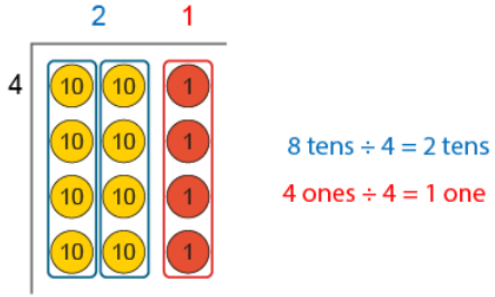
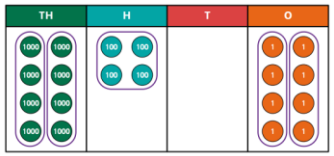
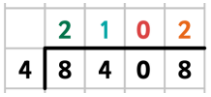
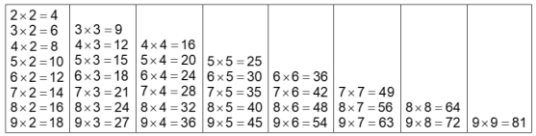
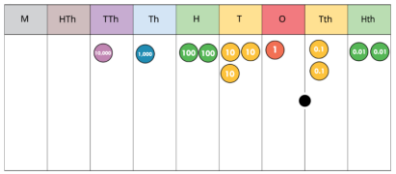
Year 5 – Division			
National curriculum	**Concrete**	**Pictorial**	**Abstract**
	Building tables  Apply tables knowledge to multiples of 10, 100 and 1000 using counting stick- forwards and backwards and with missing jumps. Create links for inverse with tables.	**Building tables**  Create links of multiplicative relationships and explore fact families to understand division facts. 4 x 12 = 48 12 x 4 = 48 48 ÷ 4 = 12 48 ÷ 12 = 4	**Building tables**  Children use understanding of known facts to help with tables I know that 7 x 7 = 49 So, I know that 49 ÷ 7 = 7 I know that 70 x 7 = 490 So, I know that 490 ÷ 7 = 70 Create fact families of known facts. Use inverse operation to find missing numbers 16 ÷ 2 = ___ 16 ÷ __ = 8 ___ x 8 = 16 ___ x 2 = 16 Understand missing number problems for division calculations and know how to solve them using inverse operations.
	Understanding factors Use concrete apparatus, such as counters to explore factors through creating arrays. Work systematically to find them all and understand that if there is a remainder it is not a factor of the given number.	**Understanding factors** Use factor diagrams to find factors. Establish common factors amongst number and work systematically to find all the factors.	**Understanding factors** Understand and explain through STEM sentences how to recognise prime and composite numbers. *Understand that prime numbers are numbers with exactly two*

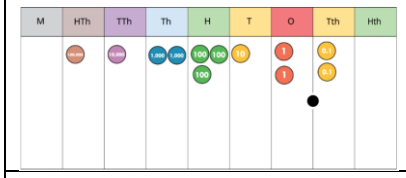
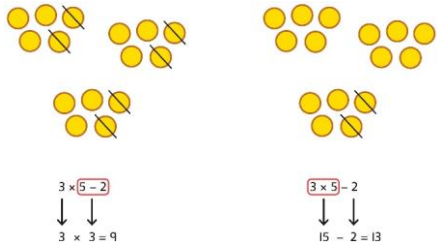
	 2 x 4 or 4 x 2 *8 ÷ 2 = 4* *8 ÷ 4 = 2* *2 and 4 are factors of 8 because they divide 8 equally with no remainders.* 5 is not a factor of 8 because 5 does not divide equally into 8.		*factors, therefore 1 is not a prime number as it only has 1 factor.* *17 ÷ 1 = 17* *17 ÷ 2 = 8 r 1* *17 ÷ 3 = 5 r 2* *17* *1 and 17 are the only factors of 17.* *17 is a prime number.* *Understand that a composite number is a number with more than 2 factors. Composite numbers are not prime numbers.*

	Known facts Use place value equipment and counters to support unitising for division by 10,100 or 1,000. Use known facts, place value and knowledge of inverse to divide mentally. *I know that 3 x 1,000= 3,000* *So, 3,000 ÷ 1,000 = 3*	**Known facts** Use a place value chart to support dividing by 10, 100 and 1,000.  *2,400 ÷ 10 = 240* Represent related facts with place value equipment when dividing by unitising. *210 is 21 tens.* *21 tens divided into groups of 3 tens. There are 7 groups.* *210 ÷ 30 = 7*	**Known facts** Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check. *6,000 ÷ 5 = 1,200* *6,000 ÷ 50 = 120* *6,000 ÷ 500 = 12* *Use the inverse to check* *5 x 1,200 = 6,000* *50 x 120 = 6,000* *500 x 12 = 6,000* Use STEM sentences to articulate understanding of why digits change when dividing by 10, 100 and 1,000. Use this knowledge to solve problems with known facts.

	Short Division Use place value resources to explore grouping for division.  	**Short Division** Use pictorial resources, such as place value charts and counters to explore short division. Ensure children know to start with the largest number and to add 0 if necessary, as a place holder. Discuss how many groups can there are in each column. 8,408 ÷ 4  Model the short division calculation alongside the pictorial resource so children start to see the connection.  Once children are secure in the method begin to work with exchanges, using pictorial methods to begin.	**Short Division** Complete the short division for up to 4-digit numbers divided by a single digit.   Children can check answers through using the inverse operation. 153 x 4 = 612 3 x 4 = 12 50 x 4 = 200 100 x 4 = 400 400 + 200 + 12 = 612
	Short Division with remainders Use concrete apparatus to understand remainders. Use multilink or concrete apparatus to group showing remainders.	**Short Division with remainders** Once children are confident with exchanging in division move onto introducing remainders. Use pictorial representations, such as place value charts. They are to understand remainders as the last remaining 1s ensuring they understand the remainder must be smaller than the divisor.	**Short Division with remainders** Children move onto problem solving contextual situations. I have 70 eggs, how many boxes will I need to buy? Each egg box can hold 6 eggs.
	Division with decimals	**Division with decimals**	**Division with decimals**

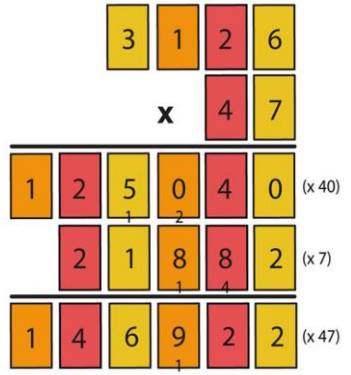
	Use place value charts and concrete apparatus to divide decimals. Understand division by 10, 100 and 1,000 using exchange and understanding which way the counters will move on a place value grid as the value becomes smaller.	Use pictorial representations to represent division showing exchange on a place value grid. Understand the movement of place value counters on place value grid, when dividing by 10 decimals become ten times smaller. Use place value grid to show.	Solve word and real-life problems involving decimals. When dividing by 10, 100 or 1,000 understand the movement on a place value grid. When dividing other amount that appear in real live context e.g., money ensure children understand the importance of the decimal. £32.80 divided by 8 people – how much does each person receive?

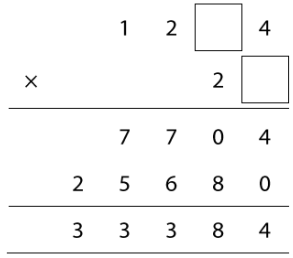
Year 6 – Multiplication			
National curriculum	**Concrete**	**Pictorial**	**Abstract**
• multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication • divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate	**Build tables**  Use counting stick activities to ensure children remain fluent in times tables, the 36 multiplication facts that are required for formal written multiplication are as follows. Children should leave year 6 knowledge of all times tables up to 12 x 12.  $2\times2=4$ $3\times2=6$ $3\times3=9$ $4\times2=8$ $4\times3=12$ $4\times4=16$ $5\times2=10$ $5\times3=15$ $5\times4=20$ $5\times5=25$ $6\times2=12$ $6\times3=18$ $6\times4=24$ $6\times5=30$ $6\times6=36$ $7\times2=14$ $7\times3=21$ $7\times4=28$ $7\times5=35$ $7\times6=42$ $7\times7=49$ $8\times2=16$ $8\times3=24$ $8\times4=32$ $8\times5=40$ $8\times6=48$ $8\times7=56$ $8\times8=64$ $9\times2=18$ $9\times3=27$ $9\times4=36$ $9\times5=45$ $9\times6=54$ $9\times7=63$ $9\times8=72$ $9\times9=81$ Use equipment to secure understanding of square numbers and cube numbers. Use facts that know to find other facts.	**Build tables**  Children create fact families from known facts – use pictorial representations with unitising. If I know 3 x 2 – I know 30 x 2 or 3,000 x 2. Understand through pictorial representations that there are multiple approaches to solving a problem. E.g., They may choose to multiply by a number by 14 through multiply by 4 and then by 10. Or multiply by 7 and then 2.	**Build tables** Children solve word problems using their times table knowledge. They use known facts to generate families of related facts. They can explain confidently which method they prefer, and which is most efficient.
	Multiplying by multiples of 10, 100 and 1,000 Use concrete apparatus, such as place value grid and counters to represent up to 7-digit numbers on a place value grid. Use place value charts and concrete apparatus to support multiplying by any of the powers of 10, including when working with decimals.  	**Multiplying by multiples of 10, 100 and 1,000** Use pictorial representations to explore multiplying by multiples, including with decimals. Discuss similarities and differences between methods, and choose efficient methods based on the specific calculation. Compare written and mental methods alongside place value representations. Derive facts from unitising and using their understanding of powers of 10. If I know :-	**Multiplying by multiples of 10, 100 and 1,000** Use efficient methods to solve word problems. Discuss various methods and confidently explain why have chosen certain method. Use what know to find missing numbers. _____ x 90,000 = 9 million

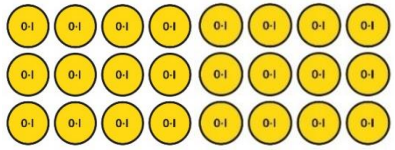
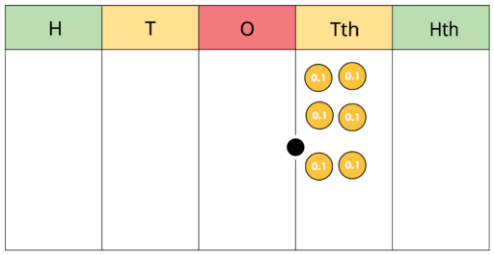
for the context • divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context • perform mental calculations, including with	11,231.22 x 10 = 112,312.2  **Order of operations** Use equipment to model different interpretations of a calculation with more than one operation. Explore different results. *3 x 5 – 2 = ?*  	4 x 2 = 8 40 x 20 = 800 400 x 200 = 80,000 4000 x 2000 = 8,000,000 **Order of operations** Model calculations using a bar model to demonstrate the correct order of operations in multi-step calculations. 12 x 4 4 4 4 4 4 4 4 4 4 4 4 4 12 x 6 6 6 6 6 6 6 6 6 6 6 6 6 Can be written as 12 x 4 + 12 x 6 48 + 72	**Order of operations** Understand the correct order of operations in calculations without brackets. Understand how brackets affect the order of operations in a calculation. Solve calculations involving varying operations.

mixed operations and large numbers • identify common factors, common multiples and prime numbers • use their knowledge of the order of operations to carry out calculations involving the 4 operations		**Multiplying up to 4 digits by 2 digits** Use an area model to break down multiplication and add each part together. Model written multiplication alongside the area multiplication. Move into long column multiplication when ready.	**Multiplying up to 4 digits by 2 digits** When children are reading move into compact column multiplication. Ensure children are secure with place value and have a secure understanding of zero as a place holder. Move into finding missing number using inverse. Solve problems involving multiplication.

	3,000	100	20	6
40	120,000	4,000	800	240
7	21,000	700	140	42





	Multiplying decimals Use concrete apparatus, such as place value counters to explore multiplications of decimals. Set out as arrays and see patterns with multiplication of integers and of decimals.  3 groups of 8 tenths is 24 tenths. 24 tenths is equal to 2 and 4 tenths. 8 groups of 3 tenths is 24 tenths.	**Multiplying decimals** Use pictorial representations, such as on a place value grid. Create links between repeated addition and multiplication of decimals. 3 rows of 0.2 3 x 0.2 = 0.6  	**Multiplying decimals** Use known facts to multiply decimals. *3 x 2 = 6* *3 x 0·2 = 0.6* Use known facts to help find missing numbers. ___ x 0.2 = 0.6

<table>
<tr><td colspan="4"><u>Year 6 – Division</u></td></tr>
<tr><td>National curriculum</td><td><u>Concrete</u></td><td><u>Pictorial</u></td><td><u>Abstract</u></td></tr>
<tr><td></td><td><u>Build tables</u>
 
Use concrete resources to link table/ division knowledge with factors and multiples.

For example, 4 is a factor of 16 but not of 19. This helps with their division recall with remainders.</td><td><u>Build tables</u>
 
Use pictorial resources to link table/ division knowledge with factors and multiples.

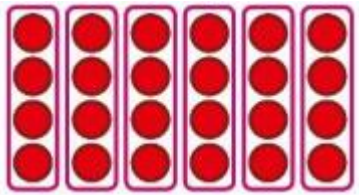
Understand the importance of knowing when a number is a composite number in division questions.

Use area models to link multiplication and division.

6 x ? = 144

<table><tr><td></td><td>10</td><td>10</td><td>2</td><td>2</td></tr><tr><td>6</td><td>60</td><td>60</td><td>12</td><td>12</td></tr></table>
6 x 24

144 ÷ 6 = 24</td><td><u>Build tables</u>
 
Use knowledge to find missing numbers.

Link fact families and known facts to help solve problems with division.</td></tr>
<tr><td></td><td><u>Dividing by single division</u>
Use equipment to create arrays to show how dividing with single division. If children cannot split into equal groups array will show remainder.
 
24 ÷ 6 = 4</td><td><u>Dividing by single division</u>
Use pictorial representations, such as a place value chart to show division. Model exchanges on place value chart and show remainder if needed. Write the short division alongside the pictorial representations.</td><td><u>Dividing by single division</u>
Use short division to solve problems.

A factory makes 98 cakes, they are to be packed in boxes of 7. How many boxes of cakes will they produce?</td></tr>
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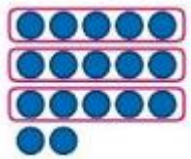
<td> 
17 ÷ 5 = 3 remainder of 2</td>
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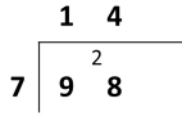

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<td><u>Long division</u>
Use pictorial representations of written process to model long division. Children will build numbers from groups using an area model alongside written process if needed.

They will use an expanded method that shows the multiples, before progressing to a more formal long division method. They divide 4-digit numbers, still without remainders, using their knowledge of multiplying by 10 and 100.

585 ÷ 15 = 39

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<tr><td>15</td><td>150</td><td>150</td><td>150</td><td>135</td></tr>
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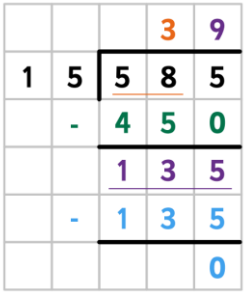
</td>
<td><u>Long division</u>
Write the required multiples to aid the division process.
15 x 1 = 15
15 x 2 = 30
15 x 3 = 45
15 x 4 = 60
15 x 5 = 75
15 x 6 = 90
15 x 7 = 105
15 x 8 = 120
15 x 9 = 135


Solve contextual problem-solving questions. These may include remainders.</td>
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<td><u>Dividing decimals</u>
Use place value equipment and concrete apparatus to divide decimals through grouping.

E.g., 0.6 ÷ 3 = 0.2

 

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<td><u>Dividing decimals</u>
Use pictorial representations to help with dividing with decimals, e.g. a place value chart or a bar model.

9.63 ÷ 3 = 3.21
 

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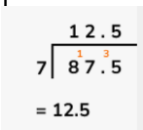
<td><u>Dividing decimals</u>
Use short division to divide decimals when appropriate.
Ensure that children do not forget the importance of the decimal place.
 

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